Fast Image Reconstruction from Non-Cartesian Data

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Introduction

MRI is an unusual medical imaging modality in that the raw data used to form an image can be collected in an infinite number of ways. While conventional Cartesian k-space scanning is by far the most common scanning method, non-Cartesian approaches have shown promise for rapid imaging, motion robustness, ultrashort TE imaging, rapid spectroscopic imaging, and other applications. A key component of implementing any non-Cartesian scanning technique is of course a method for reconstructing an image from arbitrary k-space positions. This lecture will cover fast methods for non-Cartesian image reconstruction.

It is straightforward to reconstruct data from arbitrary k-space samples using a technique known as conjugate phase reconstruction (CPR) [1, 2]. CPR is performed by multiplying each k-space point by the conjugate of the k-space encoding exponential and by a density compensation factor and then summing all of the data to arrive at a reconstructed pixel. However, this process must be repeated for each reconstructed pixel, and thus CPR is very slow. There have been a variety of methods published for fast non-Cartesian image reconstruction. Here we will focus on a technique known as gridding, which is fast, simple and widely used [3–7]. Other methods have advantages in certain situations.

The basic idea of gridding is simply to distribute the data from a non-Cartesian k-space trajectory onto a rectilinear grid, which is then be followed by an inverse fast Fourier transform (FFT) to transform to image space. The distribution is performed using a convolution operation, which can be thought of as an interpolation. The speed of gridding varies depending upon the details of the particular implementation, but roughly speaking about half of the reconstruction time is taken up by the FFT, so a simple non-Cartesian reconstruction takes about twice as long as a conventional Cartesian reconstruction.

Gridding consists of the following steps:

1. Estimate the k-space trajectory.
2. Multiply the raw data by a density compensation function (DCF).
3. Convolve the data into a rectilinear array.
4. Perform an inverse FFT.
5. Multiply by an image deapodization function that compensates for the effect of the convolution.

The following sections describe some practical considerations for each of these steps.

Trajectory Estimation

The k-space trajectory is of course a required input to any non-Cartesian image reconstruction method, including gridding. The theoretical trajectory is known by the pulse programmer, but the actual trajectory that is produced in the scanner differs from the theoretical trajectory because of a variety of factors, including eddy currents, filter-induced delays, main field inhomogeneity, susceptibility and concomitant gradients. Using an incorrect k-space trajectory can result in significant image artifacts. Thus, it is essential to either measure the trajectory [8–11] or to use some form of k-space model [12]. At a minimum, the bulk gradient delay should be measured relative to the theoretical trajectory.

Density Compensation

Conventional Cartesian k-space trajectories typically have uniform density, so each k-space point is either weighted equally or weighted with a k-space windowing function to minimize sidelobes of the point spread function (PSF). Non-Cartesian trajectories, however, often have nonuniform k-space density, and thus weighting each data point equally in the reconstruction will lead to an inaccurate reconstruction. This nonuniform k-space density can result both from varying distances between neighboring portions of the trajectory and from varying velocities along the trajectory, corresponding to varying gradient magnitude during the scan. It is important to compensate for non-uniform density to get an accurate reconstruction using CPR or gridding. (Some other non-Cartesian reconstruction methods do not require a density compensation step.)

At first glance, it would seem to be possible to compensate for the varying k-space density after convolving the data samples into the grid. It is straightforward to keep track of how much k-space "energy" is deposited in each grid point and then divide by this energy after completing the convolution. This method is referred to a post-compensation, and it can be used when the density varies slowly in space, as compared to the width of the convolution kernel. However, this step interferes with the image deapodization step and does not work well for many common k-space trajectories (e.g., spiral). Thus, it is best to perform the density compensation prior to the convolution, when it is simply a point-by-point weighting of the raw data by the DCF, which is designed to be the inverse of the local k-space density.

There are a variety of ways to calculate the DCF. For many trajectories, straightforward geometric arguments can be used to calculate the DCF. For radial scans, the problem is well-known from computed tomography and the DCF is the so-called rho filter, which is proportional to the distance from the center of k-space multiplied by the normalized gradient magnitude (if it varies during the scan). For undersampled radial scans, it may be advantageous to roll off the rho filter in the undersampled region, thus effectively apodizing k-space and reducing the aliased energy [13]. For constant-density (i.e., Archimedean) spiral scans, there are also simple DCF expressions [5, 14, 15]. These can be adapted to account for warping of k-space caused by inhomogeneity [16].

For more general trajectories, it may not be possible to generate a theoretical DCF, and thus numerical methods are necessary. For scans that have a monotonically increasing radius in k-space, including variable-density spiral scans, a simple method based on the differential area of the annulus corresponding to a particular data sample often works well [17]. A general method based on calculating the Voronoi diagram of the sample distribution works well for a wide variety of trajectories [18]. Similarly, an iterative numerical method works well for arbitrary trajectories [19], and it can be advantageous to design the DCF and the convolution kernel together [20]. One advantage of a DCF calculation method that works for any trajectory is that the actual trajectory including distortions can be used in the calculation.

Convolution and Inverse FFT

The steps of convolving onto the uniform grid and transforming into image space are related, so they are presented together here. The core idea of gridding is that the nonuniformly spaced k-space samples are convolved with a kernel that is a finite approximation to a sinc function, so that in image space the object is multiplied by a rect function approximately the width of the desired field-of-view (FOV). The result of this convolution only needs to to be evaluated at the rectilinear grid points. The data samples can be modeled as weighted delta functions, so the result of convolution with a continuous kernel function is a replica of the function at the sample point, weighted by the product of the signal amplitude and the DCF. Thus, one can think of the continuous result of the convolution as being a series of "tents", with the weighted signal amplitude corresponding to the height of the tent pole. Each of these k-space tents can then be evaluated at each grid point that falls within the finite extent of the tent, and the height of the tent at that point is then added to the resulting grid point.

The choice of kernel is one important parameter. One common choice is a Kaiser-Bessel function, which is an excellent approximation to the ideal prolate spheroidal wave function. Typically, a separable Kaiser-Bessel kernel is used, which leads to faster image reconstruction and makes sense for a square or rectangular FOV. Note that even with a Kaiser-Bessel kernel of optimal width, there will still be aliased energy at the edges of image space, because the finite extent of k-space sampling implies that the corresponding object is infinite in extent. This infinite object is then replicated by the uniform sampling corresponding to the FFT, and thus the sidelobes of the object overlap with the image FOV. The multiplication of the object by the transform of the gridding kernel mostly removes this effect, but if no room is left for a transition band, there will still be some aliasing resulting from the gridding reconstruction. Often this is acceptable, because the aliased energy will be mostly confined to the edges of the image, which may not matter for certain applications. However, for a more accurate reconstruction, it is better to do k-space oversampling.

The idea behind k-space oversampling is to use a k-space grid that is spaced more closely than would be required for a Cartesian image reconstruction. The result is to move the replications of the object in image space farther apart, and thus to allow for a transition band for the transform of the convolution kernel. The resulting extra FOV can then be discarded, similar to the phase oversampling technique often used in Cartesian image reconstruction. Beatty *et al.* have an excellent discussion of the computational and image accuracy tradeoffs involved in k-space oversampling [7]. The authors describe how the width of the Kaiser-Bessel kernel should be designed in concert with the k-space oversampling factor. An oversampling ratio of about 1.25 leads to improved image quality and a rapid image reconstruction. It is not always true that smaller oversampling ratios lead to faster image reconstructions, though, because the computation time of modern FFT algorithms (e.g., FFTW) is not monotonic with the size of the transform.

Typically, the convolution kernel is precalculated and stored in an array, which requires less computation. Linear interpolation between these samples allows fewer kernel samples to be used than are necessary with nearest neighbor interpolation for a given reconstruction accuracy [7].

Deapodization

The final step in a gridding image reconstruction is to remove the effect of the k-space convolution on the image. This effect is simply a weighting or apodization of the image by the transform of the gridding kernel. Thus, the center of the image of a uniform phantom will appear brighter than the edges. Because the analytical inverse Fourier transform of the Kaiser-Bessel kernel is known, it is a simple matter to multiply the image on a point-by-point basis by the reciprocal of this transform. The kernel should be designed so that the first zero of its transform is outside the image FOV, so that the reciprocal is defined throughout the FOV.

Summary

Gridding is a simple and rapid image reconstruction method for non-Cartesian k-space trajectories. The principal error introduced by gridding is a small amount of added aliasing energy at the edges of the image. With proper choice of parameters, it is straightforward to reduce this error to below the level of the noise in the image. Typically, the dominant sources of image artifacts in a non-Cartesian scan are from inaccuracies in the k-space trajectory estimation or from other nonidealities, especially B0 inhomogeneity. An accurate non-Cartesian reconstruction will usually include B0 inhomogeneity compensation, which will be covered in another lecture in this series.

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